2.1)

$a_1 \pm \sigma_1 \quad , \; a_2 \pm \sigma_2$

a) $$S = \left(\frac{a_1 - \bar{a}}{\sigma_1}\right)^2 + \left(\frac{a_2 - \bar{a}}{\sigma_2}\right)^2$$ to optimize:

$$\frac{\partial S}{\partial \bar{a}} = -\frac{2(a_1 - \bar{a})}{\sigma_1^2} - \frac{2(a_2 - \bar{a})}{\sigma_2^2} = 0 = \frac{\bar{a} - a_1}{\sigma_1^2} + \frac{\bar{a} - a_2}{\sigma_2^2}$$

solving for $\bar{a}$, we find

$$\bar{a} = \frac{\frac{a_1}{\sigma_1^2} + \frac{a_2}{\sigma_2^2}}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$$

$$\sigma^2 = \left(\frac{1}{2}\frac{\partial^2 S}{\partial \bar{a}^2}\right)^{-1} \Rightarrow \quad \frac{1}{\sigma^2} = \frac{1}{2}\frac{\partial^2 S}{\partial \bar{a}^2}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

b) $$L = \alpha a_1 + (1-\alpha) a_2 \qquad \sigma^2 = \left(\frac{\partial L}{\partial a_1}\right)^2 \sigma_1^2 + \left(\frac{\partial L}{\partial a_2}\right)^2 \sigma_2^2$$

minimize: $$\frac{\partial \sigma^2}{\partial \alpha} = 0 = \frac{\partial}{\partial \alpha}\left[\alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2\right]$$

$$\alpha \sigma_1^2 + (-1)(1-\alpha)\sigma_2^2 = 0 \Rightarrow \alpha = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

$$L = \frac{\sigma_2^2 a_1}{\sigma_1^2 + \sigma_2^2} + \frac{\sigma_1^2 a_2}{\sigma_1^2 + \sigma_2^2} = \frac{\frac{a_1}{\sigma_1^2} + \frac{a_2}{\sigma_2^2}}{1/\sigma_1^2 + 1/\sigma_2^2} = \bar{a}$$

$$\sigma_{min}^2 = \alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2$$
$$= \frac{\sigma_2^4 \sigma_1^2}{(\sigma_1^2 + \sigma_2^2)^2} + \frac{\sigma_1^4 \sigma_2^2}{(\sigma_1^2 + \sigma_2^2)^2} = \frac{(\sigma_2^2 + \sigma_1^2)\sigma_1^2 \sigma_2^2}{(\sigma_2^2 + \sigma_1^2)^2}$$
$$= \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} = \frac{1}{1/\sigma_1^2 + \frac{1}{\sigma_2^2}}$$

so everything is consistent