1.3) i) $P(t)\,dt = \frac{1}{\tau} e^{-t/\tau} dt \quad \text{for } t > 0$

$$\int_0^\infty P(t)\,dt = -e^{-t/\tau}\Big|_0^\infty = -0 + 1 = 1$$ as expected if normalized

ii) $$\bar{t} = \int_0^\infty t\,P(t) = \frac{1}{\tau}\int_0^\infty t\,e^{-t/\tau}\,dt$$

$$\bar{t} = \frac{1}{\tau}\left[\tau\int_0^\infty e^{-t/\tau}\,dt + t\,e^{-t/\tau}\Big|_0^\infty\right] = \int_0^\infty e^{-t/\tau}\,dt = -\tau e^{-t/\tau}\Big|_0^\infty = \tau$$

$$\bar{t} = \tau$$

iii) $$\overline{(t-\tau)^2} = \frac{1}{\tau}\int_0^\infty (t-\tau)^2 e^{-t/\tau}\,dt = \frac{1}{\tau}\int_0^\infty (t^2 - 2t\tau + \tau^2)\,e^{-t/\tau}\,dt$$

$$= \frac{1}{\tau}\left[\int_0^\infty t^2 e^{-t/\tau}\,dt - 2\tau^3 + \tau^2\int_0^\infty e^{-t/\tau}\,dt\right]$$

$$= \frac{1}{\tau}\int_0^\infty t^2 e^{-t/\tau}\,dt - \tau^2$$

$$(t-\tau)^2 = \frac{2}{\tau}\int_0^\infty t\,\tau\,e^{-t/\tau}\,dt - \tau^2 = \tau^2$$

iv) $$\frac{2.75}{10} = 0.275 = \bar{t}$$

$$\text{error} = \frac{0.275}{\sqrt{10}} = 0.09 \qquad \tau = (0.28 \pm 0.09)\times 10^{-12}\,\text{s}$$

1.4) measurements:
165.6 ± 0.3 cm
165.1 ± 0.4
166.4 ± 1.0
166.1 ± 0.8

$$\bar{x} = \frac{\sum \frac{x_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}} = \frac{\frac{165.6}{(0.3)^2} + \frac{165.1}{(0.4)^2} + \frac{166.4}{1} + \frac{166.1}{(0.8)^2}}{\frac{1}{(0.3)^2} + \frac{1}{(0.4)^2} + 1 + \frac{1}{(0.8)^2}} = \frac{3298}{19.2}$$

$$\frac{1}{\sigma^2} = 19.2 \qquad \sigma = 0.22$$

$$\bar{x} = 165.5 \pm 0.2$$

1.7) $$f = x - 2y + 3z$$

$$\sigma_f^2 = \left(\frac{\partial f}{\partial x}\right)^2\sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2\sigma_y^2 + \left(\frac{\partial f}{\partial z}\right)^2\sigma_z^2 = \sigma_x^2 + 4\sigma_y^2 + 9\sigma_z^2$$