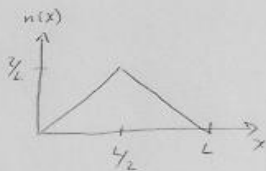


$$i) \quad n(x) = \begin{cases} \frac{4x}{L^2} & \text{for } 0 \leq x \leq \frac{L}{2} \\ \frac{4(L-x)}{L^2} & \text{for } \frac{L}{2} \leq x \leq L \\ 0 & \text{otherwise} \end{cases}$$



note, from figure, it is obvious that  $\bar{x} = \frac{L}{2}$  and  $n(x)$  is normalized properly - always make a plot!

$$\bar{x} = \int_{\text{all } x} x n(x) dx = \int_0^{L/2} \frac{4x^2}{L^2} dx + \int_{L/2}^L \frac{4(L-x)x}{L^2} dx$$

$$\bar{x} = \frac{4}{L^2} \int_0^{L/2} x^2 dx + \frac{4}{L} \int_{L/2}^L x dx - \frac{4}{L^2} \int_{L/2}^L x^2 dx$$

$$\bar{x} = \frac{L}{6} + 2L - \frac{4}{2} - \frac{4L}{2} + \frac{L}{6} = \frac{1}{2}L \quad \text{as expected!}$$

$$S^2 = \int_{\text{all } x} (x - \bar{x})^2 n(x) dx = \int_0^{L/2} \frac{4x}{L^2} (x - \frac{L}{2})^2 dx + \int_{L/2}^L \frac{4(L-x)}{L^2} (x - \frac{L}{2})^2 dx$$

$$S^2 = \frac{8}{L^2} \int_0^{L/2} x(x^2 + \frac{L^2}{4} - xL) dx$$

$$S^2 = \frac{8}{L^2} \int_0^{L/2} x^3 dx + 2 \int_0^{L/2} x dx - \frac{8}{L} \int_0^{L/2} x^2 dx$$

$$S^2 = \left(\frac{1}{8} + \frac{1}{4} - \frac{1}{3}\right)L^2 = 0.0417L^2$$

$$S = 0.204L$$

same as first term, which you can see by letting  $y = L - x$

- v) for i)  $\frac{S}{\bar{x}} = 0.648$  } similar (flat distributions)  
 ii)  $\frac{S}{\bar{x}} = 0.578$  }  
 iii)  $\frac{S}{\bar{x}} = 0.408$  } similar  
 iv)  $\frac{S}{\bar{x}} = 0.408$  } (triangular)